

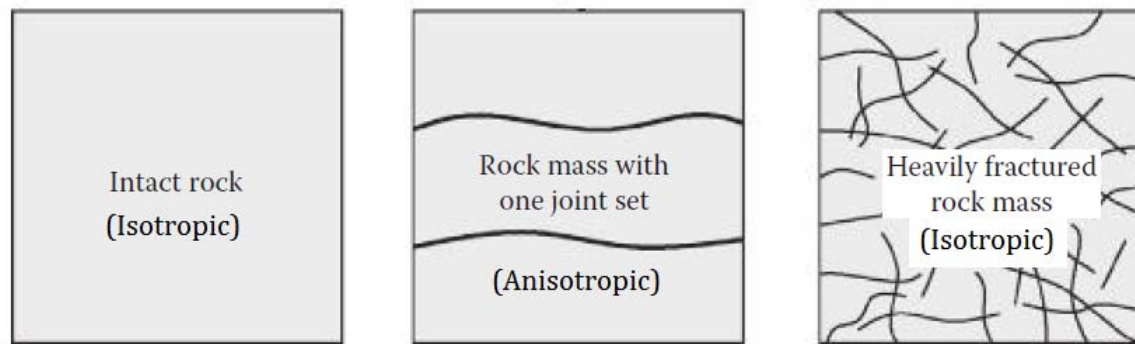
**CVG4184 / 6305: Rock Mechanics
Winter 2020**

**Chapter 5: Strength and
Deformation Characteristics
of Rocks**

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Introduction

- Major differences between soils and rocks:
 - Soils are particulate media, and rocks can be seen as a **disjointed continuum**.
 - In rocks there is a significant **scale effect**, which is not present in soils.



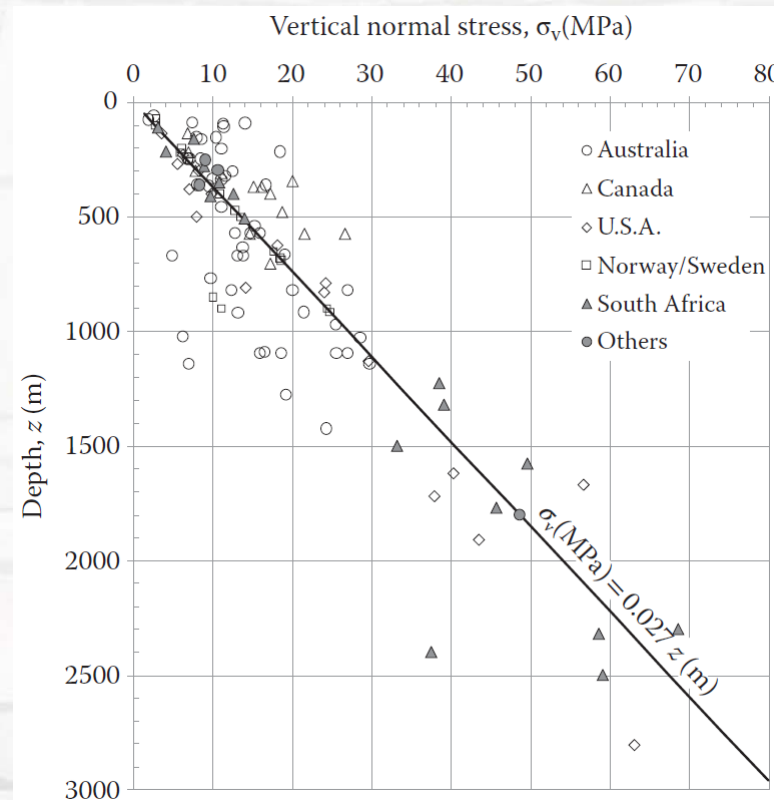
- While the intact rock can show significant **tensile strength**, the rock mass will have little or no tensile strength due to the presence of discontinuities. Soils exhibit no tensile strength.

Introduction

- Major differences between soils and rocks:
 - **Primary permeability** (through pores) of intact rock is extremely low while the rock mass may have high **secondary permeability** (through discontinuities).
 - Soils only have **primary permeability**.
 - The **strength** of soils and the intact rock **increases with the confining pressure**.
 - **Rock strength** given by **Hoek–Brown failure criterion** (the failure envelope is parabolic).
 - **Soil strength** given by the **Mohr–Coulomb failure criterion** (linear failure envelope).

In-situ stresses and strength

- **Unit weight** – commonly taken as 27 kN/m^3 .
- Vertical stress varies linearly with depth.

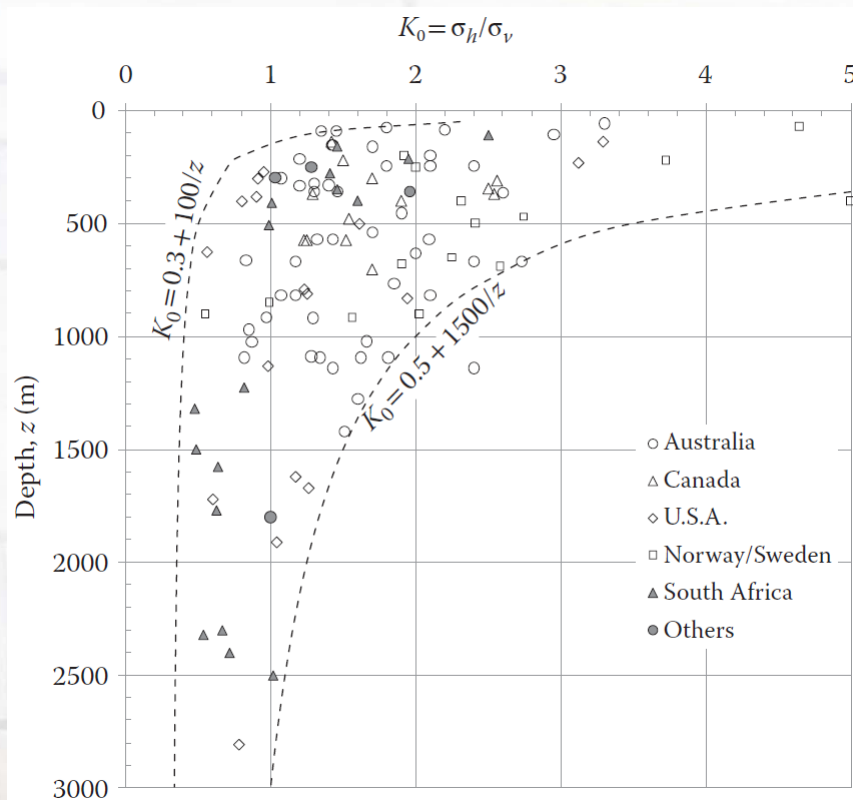


After Hoek and Brown (1980)

In-situ stresses and strength

- Horizontal **stress** is often larger than the vertical.

$$K_0 = 0.25 + 7E_h (GPa) \times \left(0.001 + \frac{1}{z(m)} \right) \quad \text{Shorey (1994)}$$



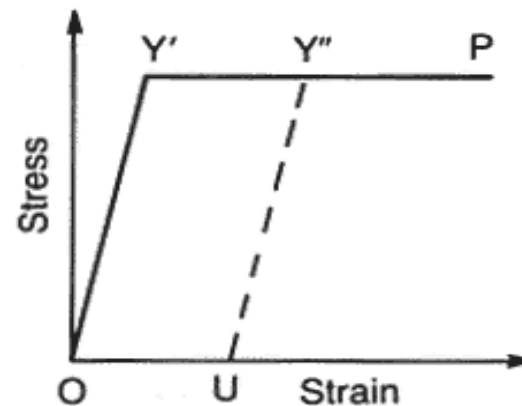
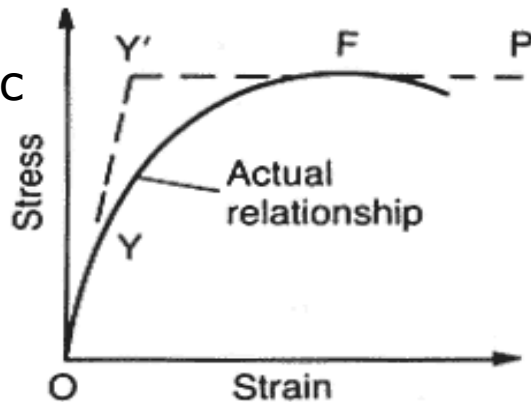
$$K_0 = \frac{\sigma_h}{\sigma_v} \geq 1.0$$

After Hoek and Brown (1980)

Stress-strain relationships

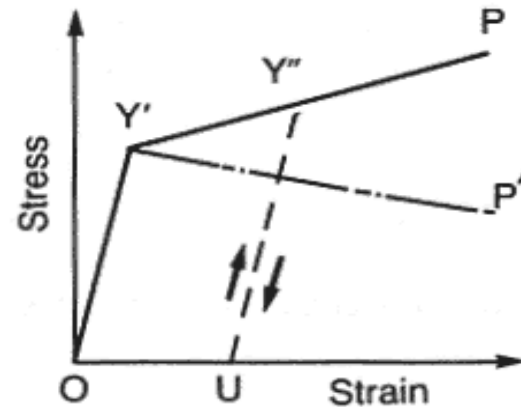
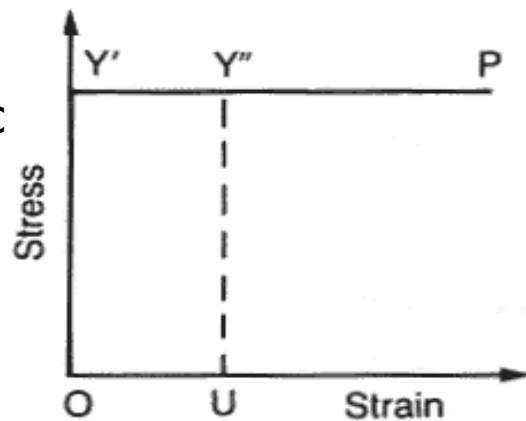
- Constitutive stress-strain models of geomaterials

Linear Elastic
 OY'



Elastic-Perfectly
Plastic

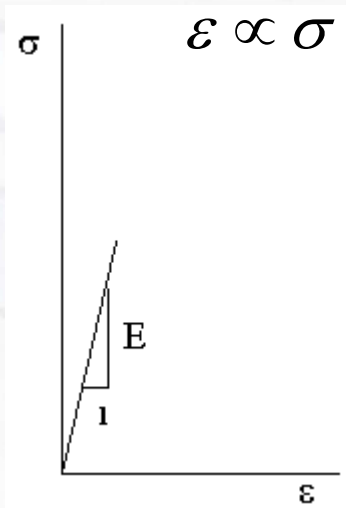
Rigid Plastic



Elastoplastic
 $Y'P$ – Strain
Hardening
 $Y'P$ – Strain
Softening

Stress-strain relationships

- The analysis of a rock mass is often carried out assuming a **linear isotropic elastic material**, following Hooke's law.



$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy}$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\gamma_{yz} = \frac{1}{G} \tau_{yz}$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{zx} = \frac{1}{G} \tau_{zx}$$

Where:

σ = normal stress; τ = shear stress;

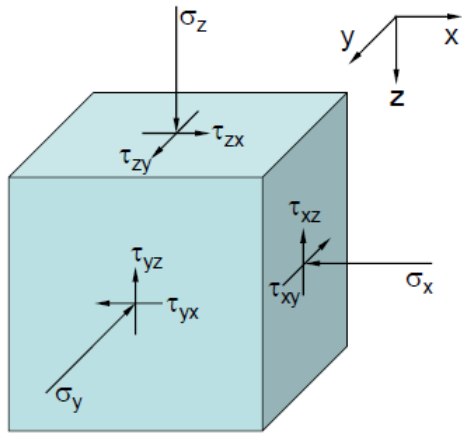
ε = normal strain; γ = shear strain;

E = modulus of elasticity; G = shear modulus;

ν = Poisson's ratio.

Stress-strain relationships

- In matrix form:



From equilibrium: $\tau_{xy} = \tau_{yx}$ $\tau_{yz} = \tau_{zy}$ $\tau_{xz} = \tau_{zx}$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

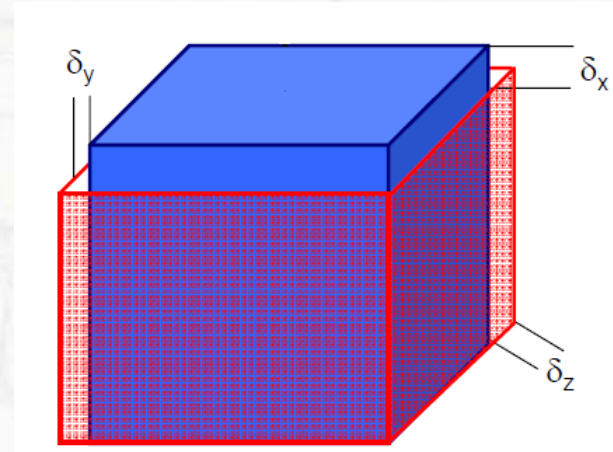
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Stress-strain relationships

- Volumetric strain, ε_{vol}

$$\varepsilon_{vol} = \frac{\delta V}{V}$$

$$\varepsilon_{vol} = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad (1)$$



Substituting the strain equations in (1) we obtain :

$$\varepsilon_{vol} = \frac{1-2\nu}{E} [\sigma_x + \sigma_y + \sigma_z]$$

$$\varepsilon_{vol} = \frac{3(1-2\nu)}{E} \left[\frac{\sigma_x + \sigma_y + \sigma_z}{3} \right] = \frac{1}{K} \left[\frac{\sigma_x + \sigma_y + \sigma_z}{3} \right]$$

Stress-strain relationships

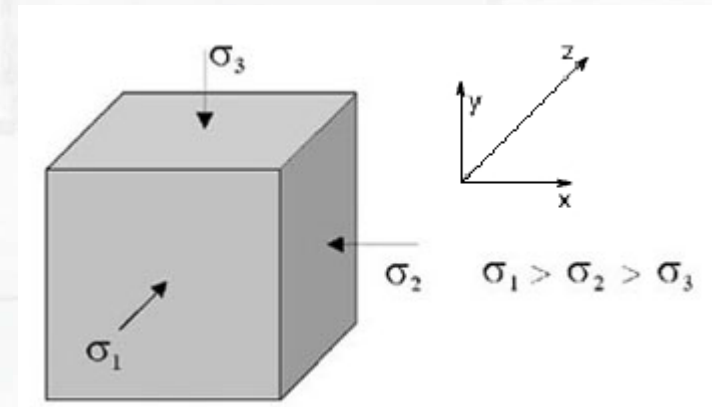
- Relationship between **elastic parameters**:

$$E = \frac{9KG}{3K + G} \quad \nu = \frac{3K - 2G}{2(3K + G)} \quad G = \frac{E}{2(1 + \nu)}$$

- Principal** stresses and strains

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & \nu \\ \nu & 1 - \nu & \nu \\ \nu & \nu & 1 - \nu \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}$$



Where: E = modulus of elasticity; G = shear modulus;
 K = bulk modulus; ν = Poisson's ratio.

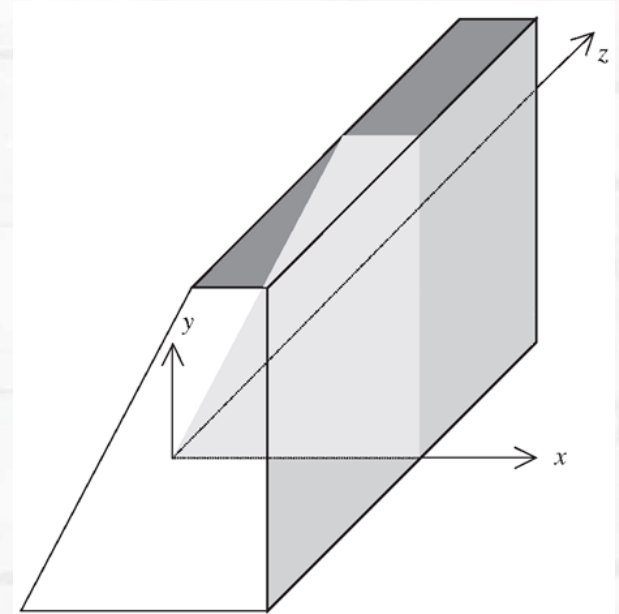
Stress-strain relationships

- Plane strain loading:** when the structure is long in one direction, the deformation or strain in this direction can be neglected.

$$\tau_{yz} = \tau_{zx} = \varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0 \quad \text{and} \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1-\nu^2 & -\nu(1+\nu) & 0 \\ -\nu(1+\nu) & 1-\nu^2 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



Stress-strain relationships

Example 1

In a rock mass subjected to plane strain loading, the principal stresses $\sigma_1 = 2$ MPa and $\sigma_3 = 1$ MPa are acting in the x and y -directions, respectively. Assuming a Young's modulus of 20 GPa and Poisson's ratio of 0.2, determine the principal strains and the normal stress perpendicular to the plane of loading.

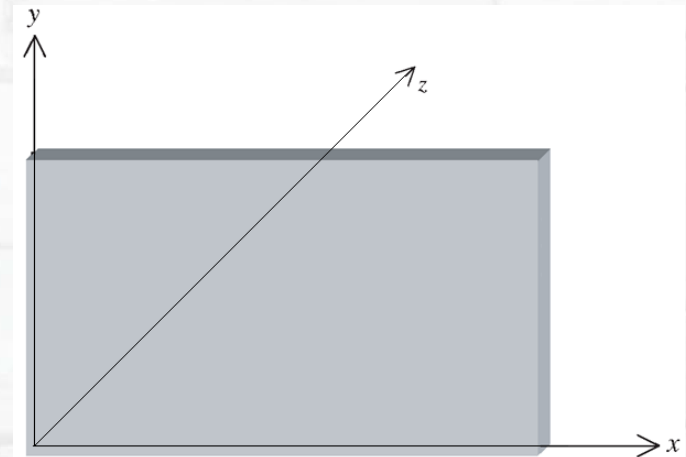
Stress-strain relationships

- **Plane stress loading**: when the structure is small in one direction (z) and the stresses are confined to the x - y plane.

$$\sigma_z = \tau_{yz} = \tau_{zx} = \gamma_{yz} = \gamma_{zx} = 0 \quad \text{and} \quad \varepsilon_z = \frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



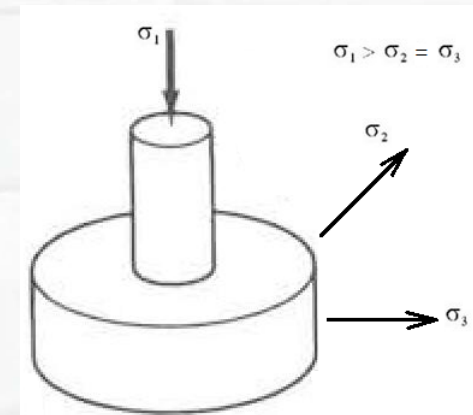
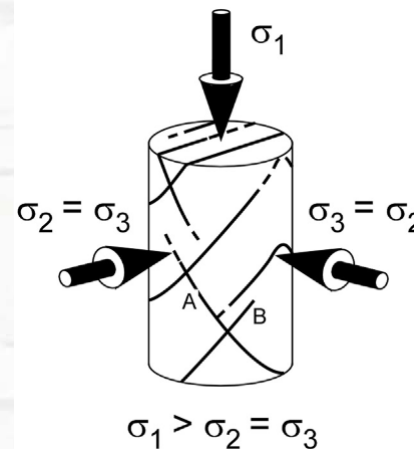
Stress-strain relationships

- **Axisymmetric loading**: when the loading is symmetrical about an axis (e.g., under a uniformly loaded circular footing or during triaxial testing).

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -2\nu \\ -\nu & 1-\nu \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & 2\nu \\ \nu & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix}$$

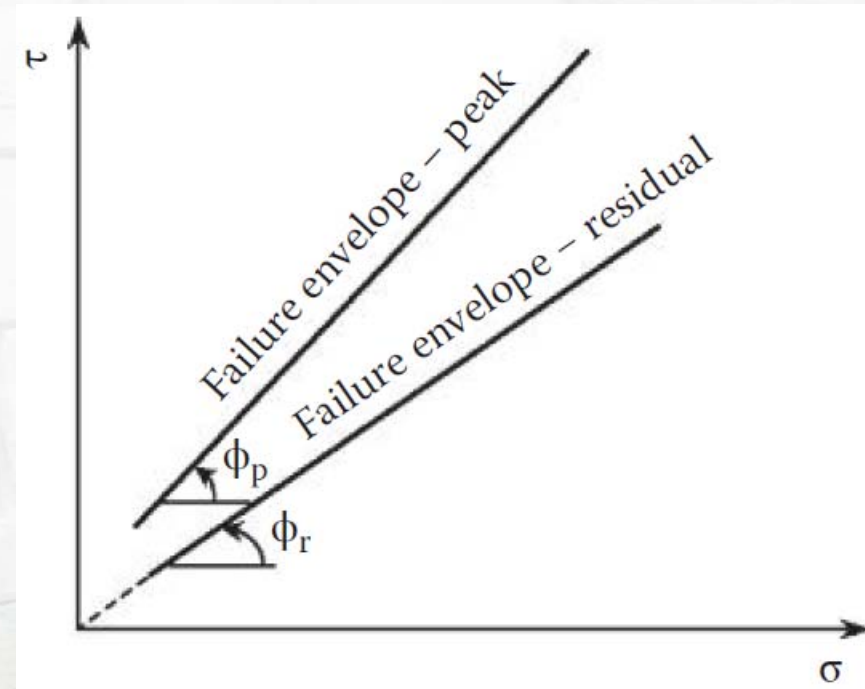
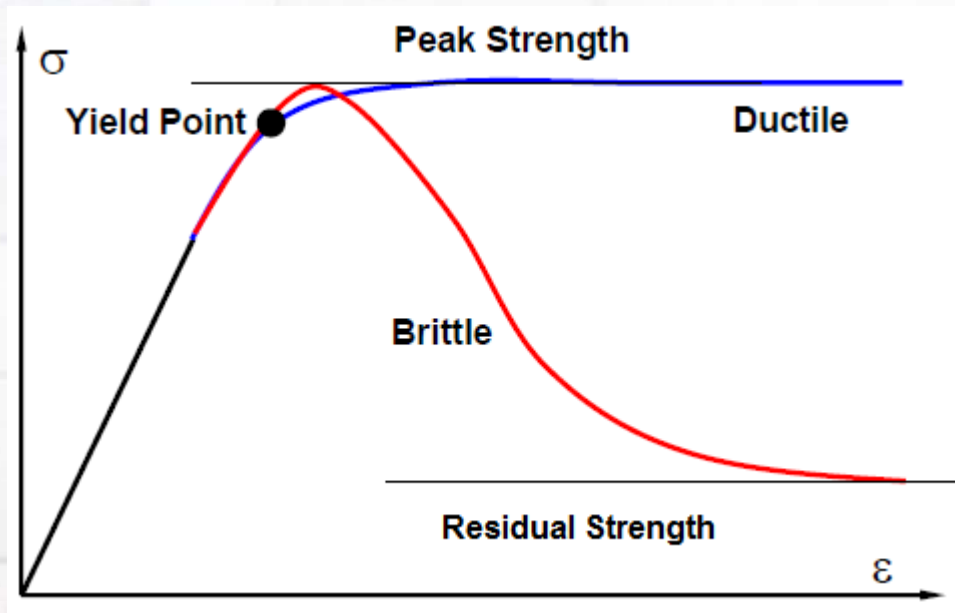


Strength and failure criteria

- **Strength** of a material is the limit state of stress the material can sustain.
- Strength is often considered in terms of compressive strength, tensile strength, and shear strength.
- Strain at the limit state of stress is often defined as **strain at failure**.
- **Yield point** defines the onset of plastic deformation.
- **Brittle** behaviour indicates a substantial reduction of stress-carrying capacity, reflecting fracturing of the material under stress.
- **Ductility** is the physical property of being capable of sustaining large plastic deformations without fracture.

Strength and failure criteria

- The shear strength of a rock mass can be defined in terms of **peak** or **residual** stresses.
- **Peak shear strength** is the maximum shear stress that can be carried by the element; **residual shear strength**, is the shear stress at large strains.



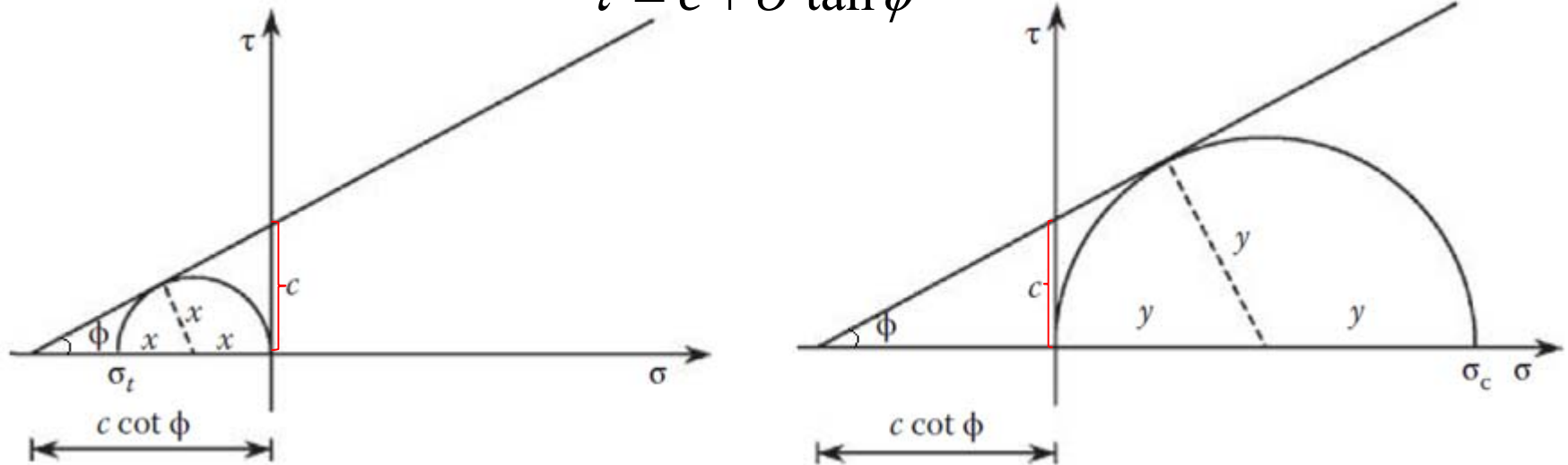
Strength and failure criteria

- The Mohr-Coulomb failure criterion for intact rock

$$\sigma_t = \frac{2c \cos \phi}{1 + \sin \phi}$$

$$\sigma_c = \frac{2c \cos \phi}{1 - \sin \phi}$$

$$\tau = c + \sigma \tan \phi$$



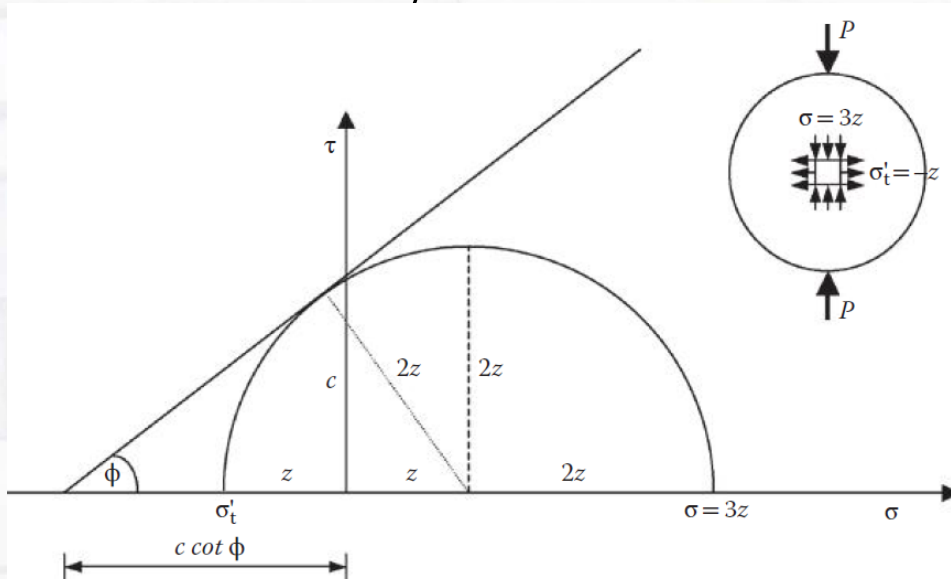
Mohr circles at failure for a) uniaxial tensile strength test; and b) uniaxial compressive strength test.

Note: As the intact rock is relatively impervious, the Mohr-Coulomb failure criterion is applied in terms of total strength parameters.

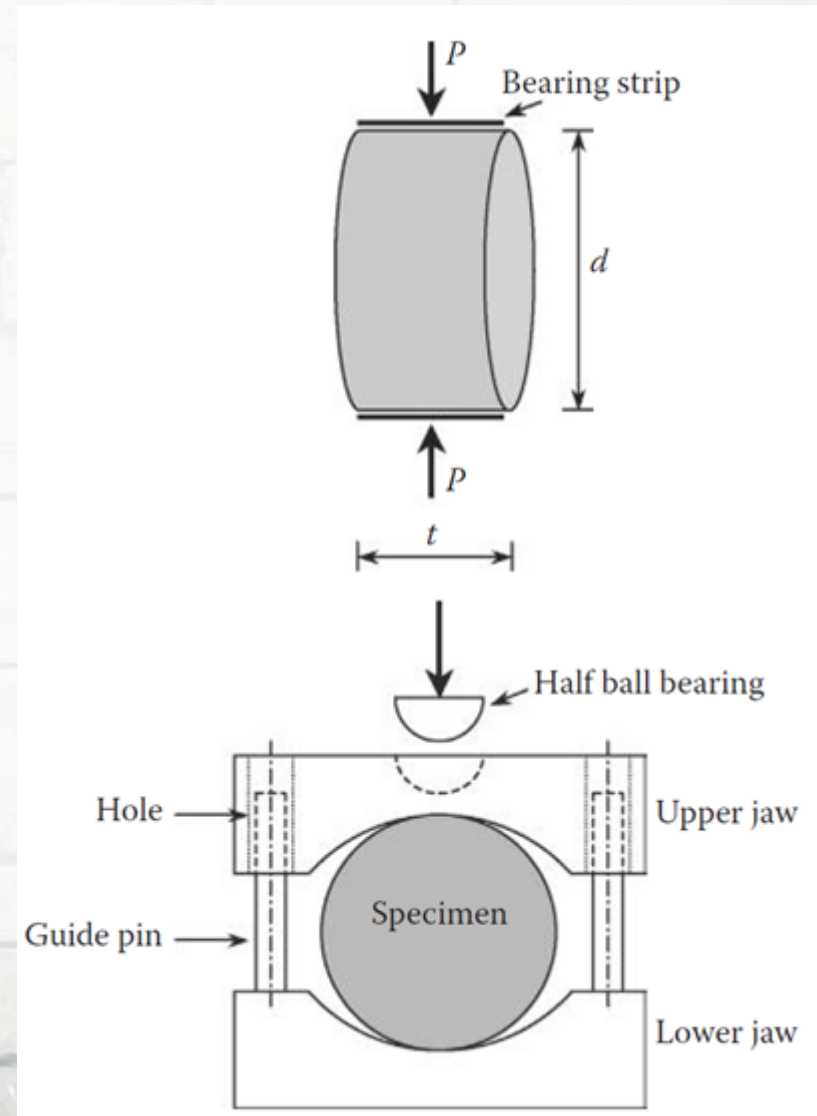
Strength and failure criteria

- The Mohr-Coulomb failure criterion for intact rock

$$\sigma'_t = \frac{c \cos \phi}{2 - \sin \phi}$$

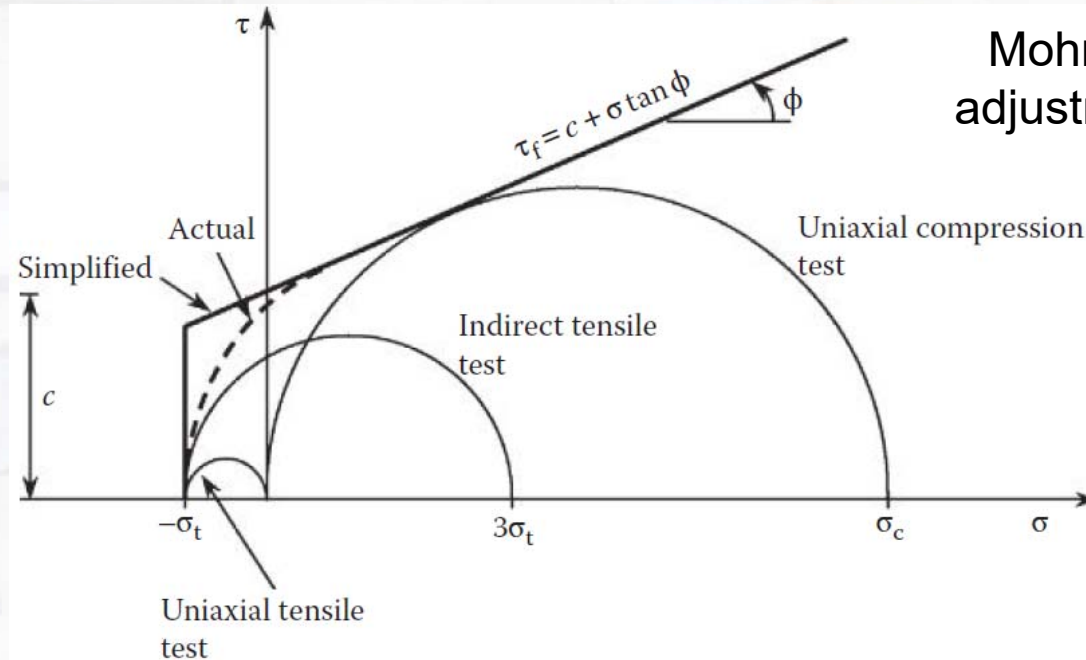


Mohr circle at failure for Brazilian indirect tensile strength test.



Strength and failure criteria

- The Mohr-Coulomb failure criterion for intact rock



Mohr-Coulomb failure criterion with adjustment for tensile normal stresses.

$$\sigma_c = 2c \tan \left(45 + \frac{\phi}{2} \right) = 2c \frac{\cos \phi}{1 - \sin \phi} = 2c \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$$

At failure:

$$\sigma_1 = 2c \tan \left(45 + \frac{\phi}{2} \right) + \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right) = \sigma_c + \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right)$$

Strength and failure criteria

Example 2

A series of six triaxial tests were carried out on 50 mm-diameter limestone cores and the following data were obtained for the principal stresses at failure:

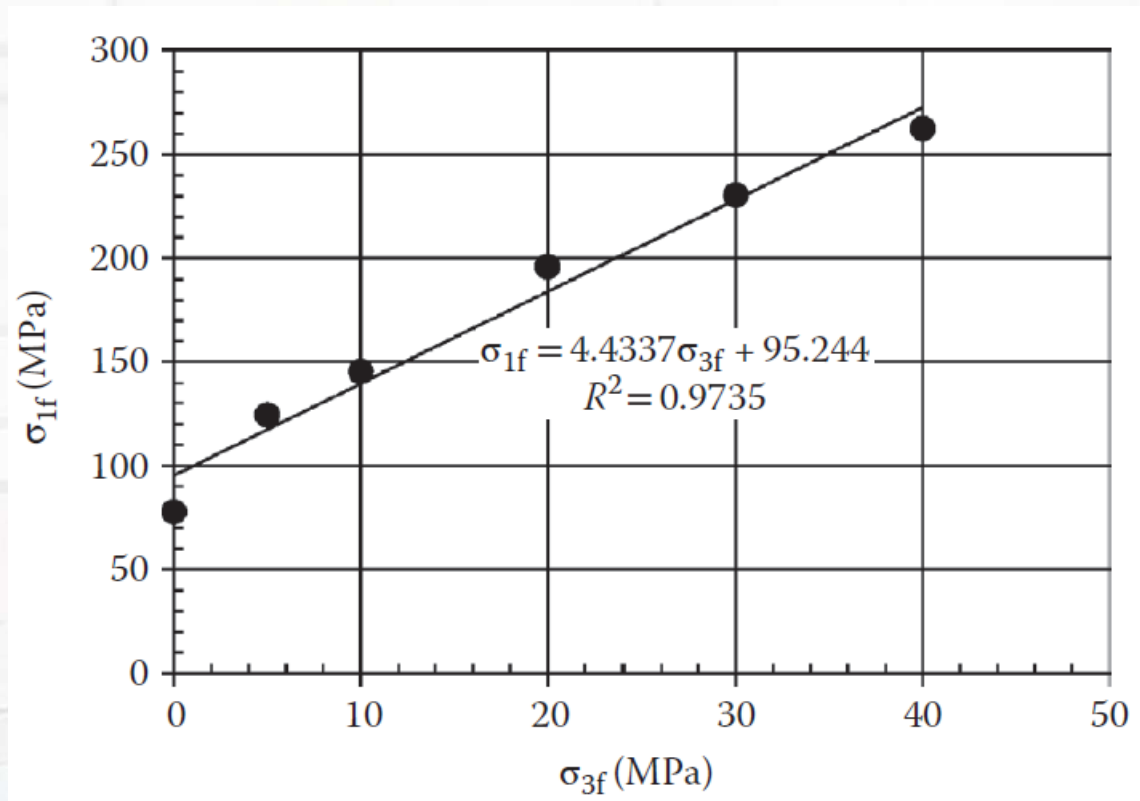
Specimen	1	2	3	4	5	6
σ_{1f} (MPa)	78.0	124.5	145.5	196.0	230.5	262.5
σ_{3f} (MPa)	0	5.0	10.0	20.0	30.0	40.0

Plot σ_{1f} vs. σ_{3f} and determine the uniaxial compressive strength, σ_c and friction angle, ϕ of the limestone.

Strength and failure criteria

Example 2 - Solution

$$\sigma_1 = \tan^2\left(45 + \frac{\phi}{2}\right)\sigma_3 + \sigma_c$$



Strength and failure criteria

Example 3

It is proposed to excavate a horseshoe-shaped tunnel at a depth of 100 m below ground level into a sound, unjointed fresh granite with $c = 0.5$ MPa and $\phi = 40^\circ$. The average unit weight of the overburden is 27 kN/m^3 . Once the tunnel is excavated, it is expected that the lateral normal stress near the tunnel walls will be close to zero. Will the rock fail ('burst') into the excavated tunnel? What level of prestressing (i.e., σ_3) in the form of struts, rock bolting and so on is required to ensure that the tunnel can just resist the failure?

Strength and failure criteria

- The Mohr-Coulomb failure criterion for rock mass

- The discontinuities present within the rock mass allow easy access to water and pore water pressures develop within the joints:

$$\sigma' = \sigma - u$$

- At failure:

$$\sigma'_1 = 2c' \tan\left(45 + \frac{\phi'}{2}\right) + \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) = \sigma_{cm} + \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$$

Where σ_{cm} is the uniaxial compressive strength of the rock mass.

Strength and failure criteria

- The Hoek-Brown failure criterion for intact rock
 - Introduced to provide input data for the analyses required for the design of underground excavations in hard rock.
 - Derived from the research into the brittle failure of intact rock by Hoek (1968) and on model studies of jointed rock mass behaviour by Brown (1970).
 - The criterion uses the properties of intact rock as input parameters, and then introduces factors to reduce these properties on the basis of the characteristics of joints in a rock mass.
 - The generalized form of the criterion can account for the disturbance in the rock mass due to blasting and stress relief.

Strength and failure criteria

- The Hoek-Brown failure criterion for intact rock

- In rock mechanics, the common practice is to present the **failure criteria** in terms of the **principal stresses** σ_3 (on the x-axis) and σ_1 (on the y-axis).
- **Hoek and Brown (1980)** proposed that the effective major, σ'_1 and minor, σ'_3 principal stresses at failure can be related by:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_i \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{0.5}$$

Where:

σ_{ci} is the uniaxial compressive strength of the intact rock;

m_i and s are material constants, where $s = 0$ for poor rock; and $s = 1$ for intact rock.

$$(\sigma'_1 - \sigma'_3)^2 = m_i \sigma_{ci} \sigma'_3 + \sigma_{ci}^2$$

Strength and failure criteria

- Parameter m_i for the Hoek-Brown failure criterion (intact rock)

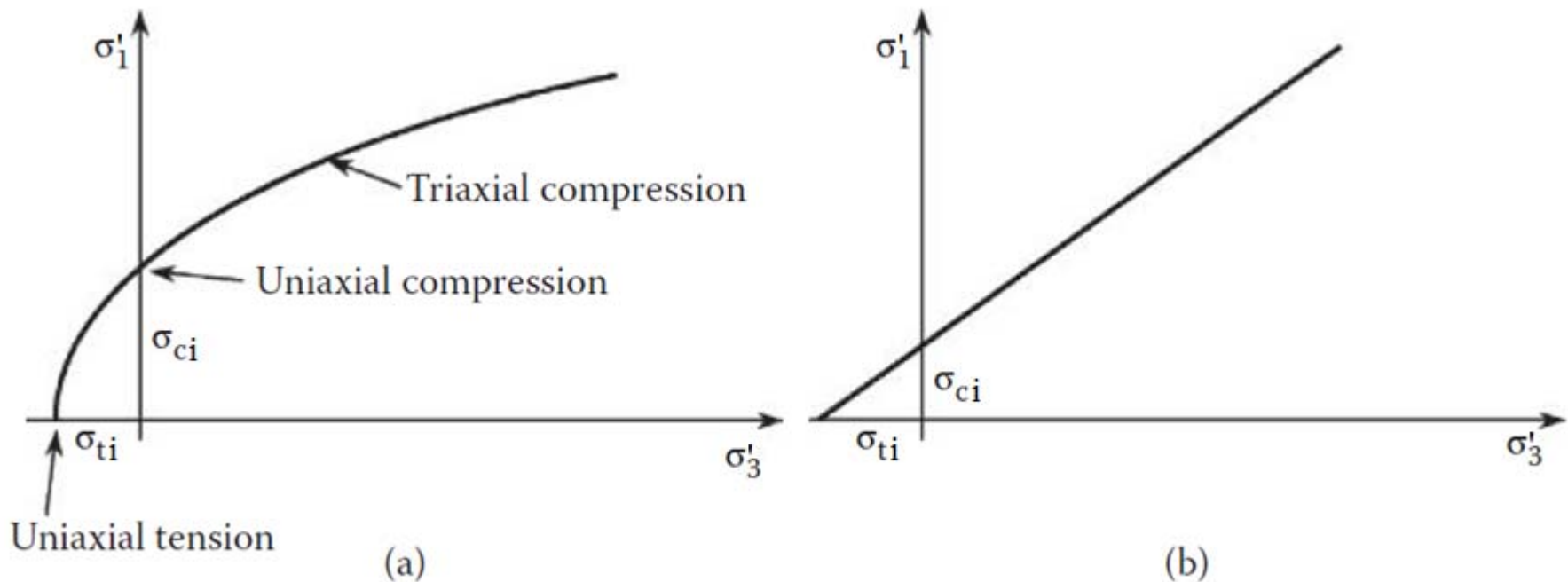
m_i values for rocks				
	Texture			
	Coarse	Medium	Fine	Very fine
Sedimentary	Conglomerates (21 ± 3)	Sandstones, 17 ± 4	Siltstones, 7 ± 2	Claystones, 4 ± 2
	Breccias (19 ± 5)		Greywackes (18 ± 3)	Shales (6 ± 2) Marls (7 ± 2)
	Crystalline limestone (12 ± 3)	Sparitic limestone (10 ± 2)	Micritic limestone (9 ± 2)	Dolomite (9 ± 3)
		Gypsom, 8 ± 2	Anhydrite, 12 ± 2	Chalk, 7 ± 2
Metamorphic	Marble, 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzite, 20 ± 3	
	Migmatite (29 ± 3)	Amphibiolites, 26 ± 6	Gneiss, 28 ± 5	
		Schists, 12 ± 3	Phyllites (7 ± 3)	Slates, 7 ± 4
Igneous	Granite (32 ± 3)	Diorite (25 ± 5)		
		Granodiorite (29 ± 3)		
	Gabbro, 27 ± 3	Dolerite (16 ± 5)		
	Norite, 20 ± 5			
	Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
		Rhyolite (25 ± 5)	Dacite (25 ± 3)	Obsidian (19 ± 3)
		Andesite, 25 ± 5	Basalt (25 ± 5)	
	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

Source: Hoek, E., and Brown, E.T., *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, 34, 1165, 1997.

Note: The values in parenthesis are estimates. The others are measured.

Strength and failure criteria

- Comparison between the Hoek-Brown and the Mohr-Coulomb failure criteria



Comparison between (a) Hoek–Brown failure criterion; and (b) Mohr–Coulomb failure criterion.

$$\left(\frac{\sigma_{ti}}{\sigma_{ci}} \right) = -\frac{\sqrt{m_i^2 + 4s} - m_i}{2} \quad \left(\frac{\sigma_{ci}}{\sigma_{ti}} \right) \approx m_i$$

Strength and failure criteria

Example 4

A series of six triaxial tests were carried out on 50 mm-diameter limestone cores and the following data were obtained for the principal stresses at failure:

Specimen	1	2	3	4	5	6
σ_{1f} (MPa)	78.0	124.5	145.5	196.0	230.5	262.5
σ_{3f} (MPa)	0	5.0	10.0	20.0	30.0	40.0

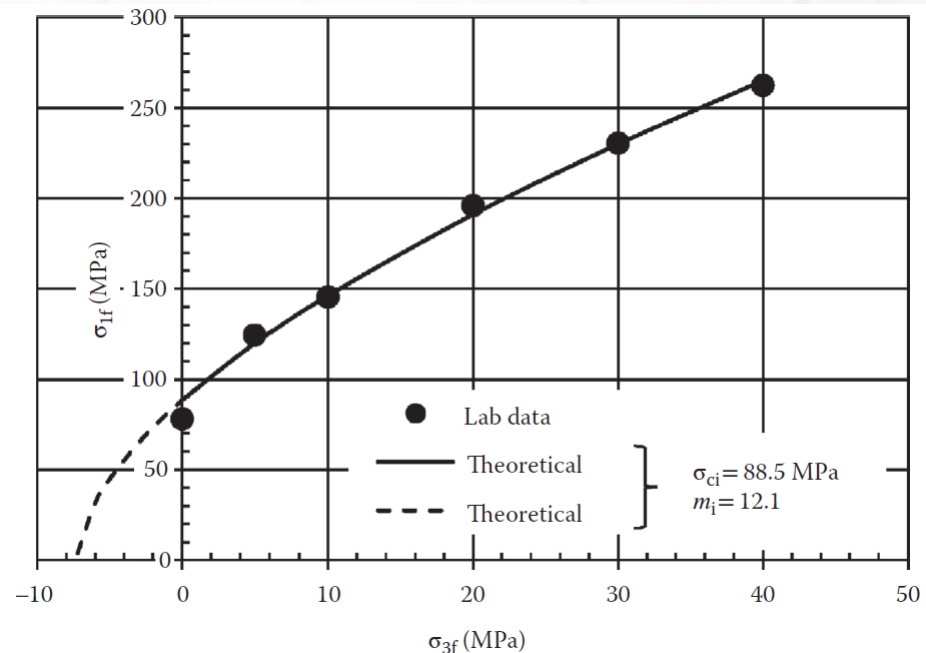
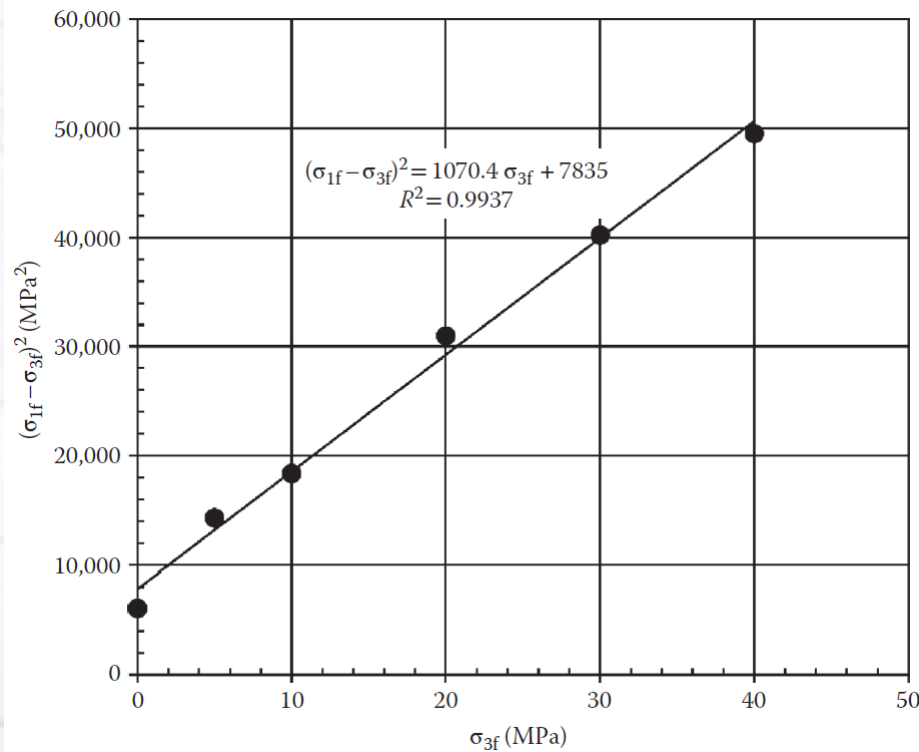
- Neglect the pore water pressures and determine m_i , σ_{ci} and σ_{ti} for the limestone.
- Using the values of m_i and σ_{ci} from (a), plot the theoretical Hoek-Brown failure envelope.
- Show the test data along with the theoretical failure envelope and comment on how well they match.

Strength and failure criteria

Example 4 - Solution

$$(\sigma'_1 - \sigma'_3)^2 = m_i \sigma_{ci} \sigma'_3 + \sigma_{ci}^2$$

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_i \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{0.5}$$



Strength and failure criteria

- The generalized Hoek-Brown failure criterion
 - Applicable to **rock mass** as well as **intact rock**.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_m \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a$$

- The parameter m_m for the rock mass is related to m_i for intact rock by:

$$m_m = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$

Where:

σ_{ci} is the uniaxial compressive strength of the intact rock;

m_m (rock mass), m_i (intact rock), a and s are material constants;

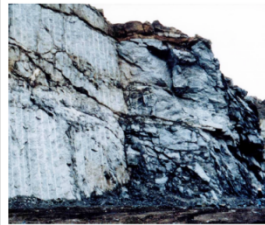
D is a factor to account for the disturbance in the rock mass due to blasting and stress relief, where $D = 0$ for undisturbed, and $D = 1$ for highly disturbed rock mass;

GSI is the Geological strength index.

Strength and failure criteria

- Disturbance factor, D for the Hoek-Brown failure criterion

Appearance of rock mass	Description of rock mass	Suggested value of D
	Excellent quality controlled blasting or excavation by Tunnel Boring Machine results in minimal disturbance to the confined rock mass surrounding a tunnel.	$D = 0$
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass. Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	$D = 0$ $D = 0.5$ No invert
	Very poor quality blasting in a hard rock tunnel results in severe local damage, extending 2 or 3 m, in the surrounding rock mass.	$D = 0.8$

Appearance of rock mass	Description of rock mass	Suggested value of D
	Small scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left hand side of the photograph. However, stress relief results in some disturbance.	$D = 0.7$ Good blasting $D = 1.0$ Poor blasting
	Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal. In some softer rocks excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.	$D = 1.0$ Production blasting $D = 0.7$ Mechanical excavation

Strength and failure criteria

- **Material constants** for the generalized Hoek-Brown failure criterion

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \quad \text{Hoek et al. (2002)}$$

Where:

$s = 0$ for poor quality rock, and $s = 1$ for intact rock;

$a = 0.50$ for intact rock, and $a = 0.65$ for poor quality rock.

GSI is the Geological strength index.

- **Uniaxial compressive strength of the rock mass, σ_{cm}**

$$\sigma_{cm} = \sigma_{ci} S^a$$

$$\sigma_{cm} = \sigma_{ci} \times 0.0034 m_i^{0.8} \times \left(1.029 + 0.025 e^{-0.1 m_i}\right)^{GSI} \quad \text{Marinos and Hoek (2001)}$$

As GSI approaches 100 (intact rock) the ratio $\sigma_{cm} / \sigma_{ci}$ approaches 1.

Strength and failure criteria

- Uniaxial tensile strength of the rock mass, σ_{tm}

$$\sigma_{tm} = -\sigma_{ci} \frac{\sqrt{m_m^2 + 4s} - m_m}{2} \quad \text{Hoek and Brown (1997)}$$

$$\sigma_{tm} = -\frac{s\sigma_{ci}}{m_m} \quad \text{Hoek et al. (2002)}$$

Where:

σ_{tm} is the uniaxial tensile strength of the rock mass;

σ_{ci} is the uniaxial compressive strength of the intact rock;

m_m and s are material constants, where $s = 0$ for poor rock; and $s = 1$ for intact rock.

$a = 0.50$ for intact rock is assumed.

Strength and failure criteria

- Typical values of Hoek-Brown parameters

Description	Intact Rock		Rock Mass				E_m (GPa)
	σ_d (MPa)	m_i	GSI	m_m	s	a	
Massive (almost intact) but weak cemented breccias – similar to weak concrete	51	16.3	75	6.68	0.062	0.501	15.0
Massive strong rock gneiss, with very few joints	110	28	75	11.46	0.062	0.501	45.0
Average quality rock mass: jointed quartz mica schist	30	15	65	4.3	0.02	0.5	10.0
Poor quality rock mass at shallow depth: Athenian schist	5–10	9.6	20	0.55	0.0001	0.544	0.60
Poor quality rock mass under high stress: 25-km-long water supply tunnel 1200 m below surface; graphitic phyllite, squeezing ground	50	10	25	0.48	0.0002	0.53	1.0

Source: Hoek, E., *Practical Rock Engineering*, http://www.rocscience.com/hoek/corner/practical_rock_engineering.pdf, 2007.

Strength and failure criteria

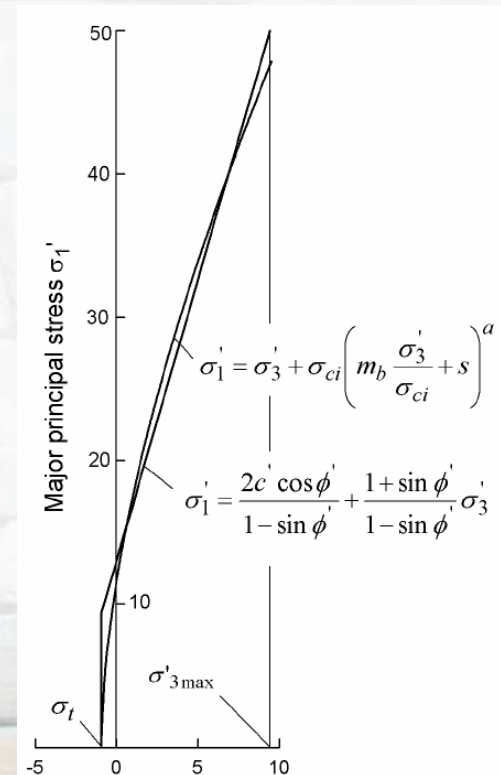
Example 5

An intact sample of massive strong rock gneiss, with very few joints, was tested in a laboratory and the following results were obtained: $GSI=75$, $m_i=28$, $\sigma_{ci}=110\text{MPa}$. Estimate:

- the rock mass parameters m_m , s and a .
- the uniaxial compressive strength of the rock mass, σ_{cm}
- How do the estimated values compare with the reported experimental data?

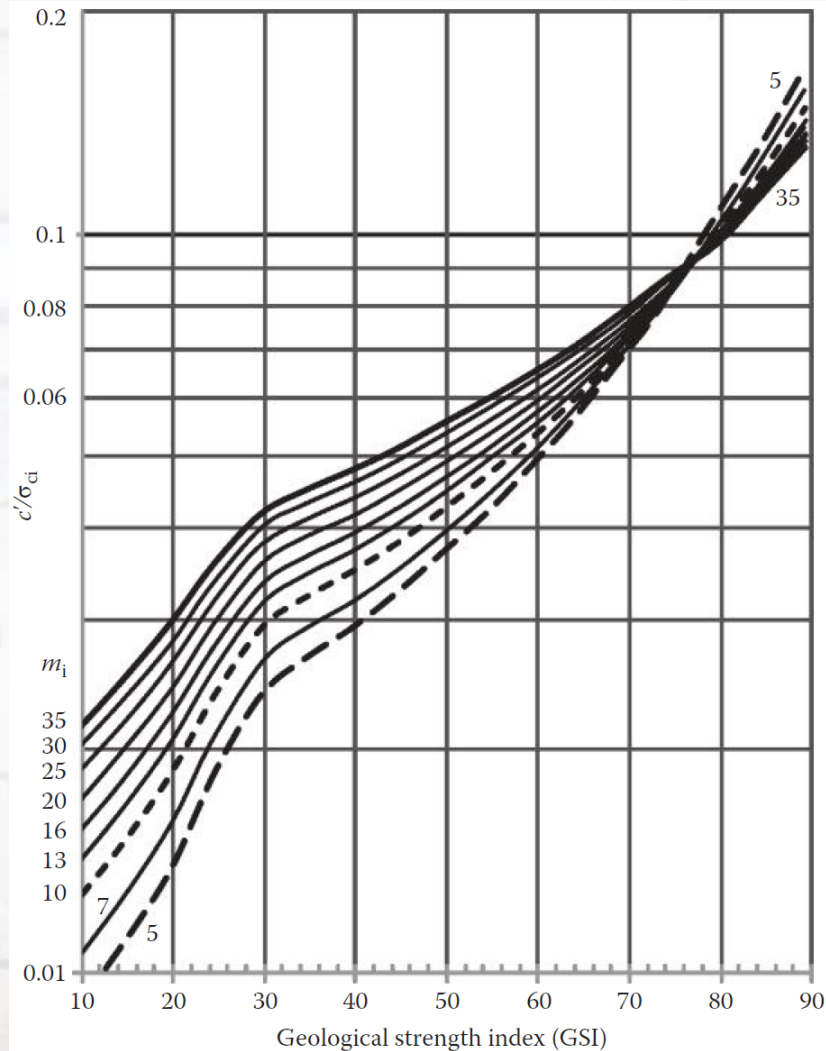
Mohr-Coulomb parameters for rock mass

- Many geotechnical problems (e.g., slope stability) are more conveniently dealt with in terms of **shear and normal stresses** rather than the **principal stresses**.
- The main difficulty is to derive the Mohr-Coulomb shear strength parameters **c' and ϕ' for the rock mass**.
- Hoek and Brown (1997) simulated a series of triaxial test data for rock masses of different GSI , m_i and σ_{ci} and plotted the Mohr-Coulomb failure envelopes, from which the **c' and ϕ'** for the rock mass were determined.

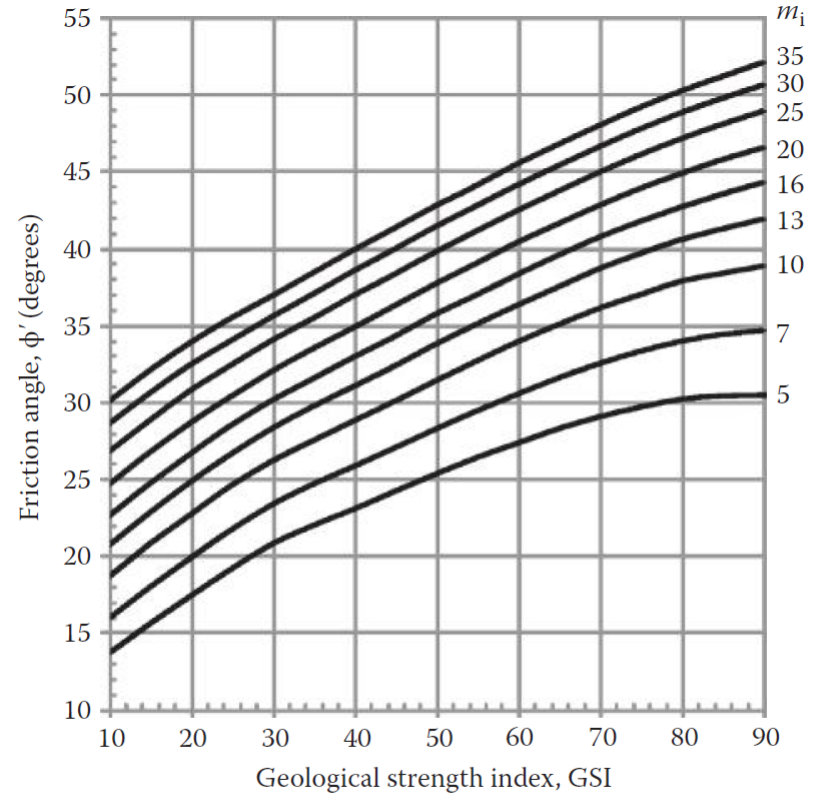


Mohr-Coulomb parameters for rock mass

- Values of c' and ϕ' for rock mass



c'/σ_{ci} – m_i –GSI relationship for the rock mass. (Hoek, E. and E.T. Brown, *Int. J. Rock Mech. Min. Sci.*, 34, 1165–1186, 1997.)



ϕ' – m_i –GSI relationship for the rock mass. (Hoek, E. and E.T. Brown, *Int. J. Rock Mech. Min. Sci.*, 34, 1165–1186, 1997.)

Mohr-Coulomb parameters for rock mass

- Values of c' and ϕ' for rock mass (Hoek et al., 2002)
- Applicable if $0 < \sigma'_3 < 0.25\sigma_{ci}$

$$\sin \phi' = \frac{6am_m(s + m_m\sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6am_m(s + m_m\sigma'_{3n})^{a-1}}$$

$$c' = \frac{\sigma_{ci}[(1+2a)s + (1-a)m_m\sigma'_{3n}](s + m_m\sigma'_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1 + [6am_m(s + m_m\sigma'_{3n})^{a-1}] / [(1+a)(2+a)]}}$$

$$\sigma'_{3n} = \sigma'_{3,\max} / \sigma_{ci}$$

Where:

σ_{ci} is the uniaxial compressive strength of the intact rock;

$\sigma'_{3,\max}$ is the upper limit of σ'_3 and depends on the project and stress level;

m_m , s and a are material constants;

c' and ϕ' are the effective shear strength parameters for the rock mass.

Mohr-Coulomb parameters for rock mass

- Determining $\sigma'_{3,\max}$ (Hoek et al., 2002)

$$\frac{\sigma'_{3,\max}}{\sigma'_{cm}} = 0.47 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.94} \quad \text{- For tunnels and underground excavations}$$

$$\frac{\sigma'_{3,\max}}{\sigma'_{cm}} = 0.72 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.91} \quad \text{- For slopes}$$

$$\sigma'_{cm} = \frac{2c' \cos \phi'}{1 - \sin \phi'}$$

Where:

σ'_{cm} is the global rock mass strength determined from the Mohr-Coulomb envelope;

c' and ϕ' are the effective shear strength parameters for the rock mass;

γ is the unit weight of the rock mass;

H is the depth of the tunnel (excavation) below surface or height of the slope.

Mohr-Coulomb parameters for rock mass

Example 6

Five intact samples of massive strong rock gneiss, with very few joints, were tested in a laboratory and the following results were obtained: $GSI=75$, $m_i=28$ and $\sigma_{ci}=110$ MPa and:

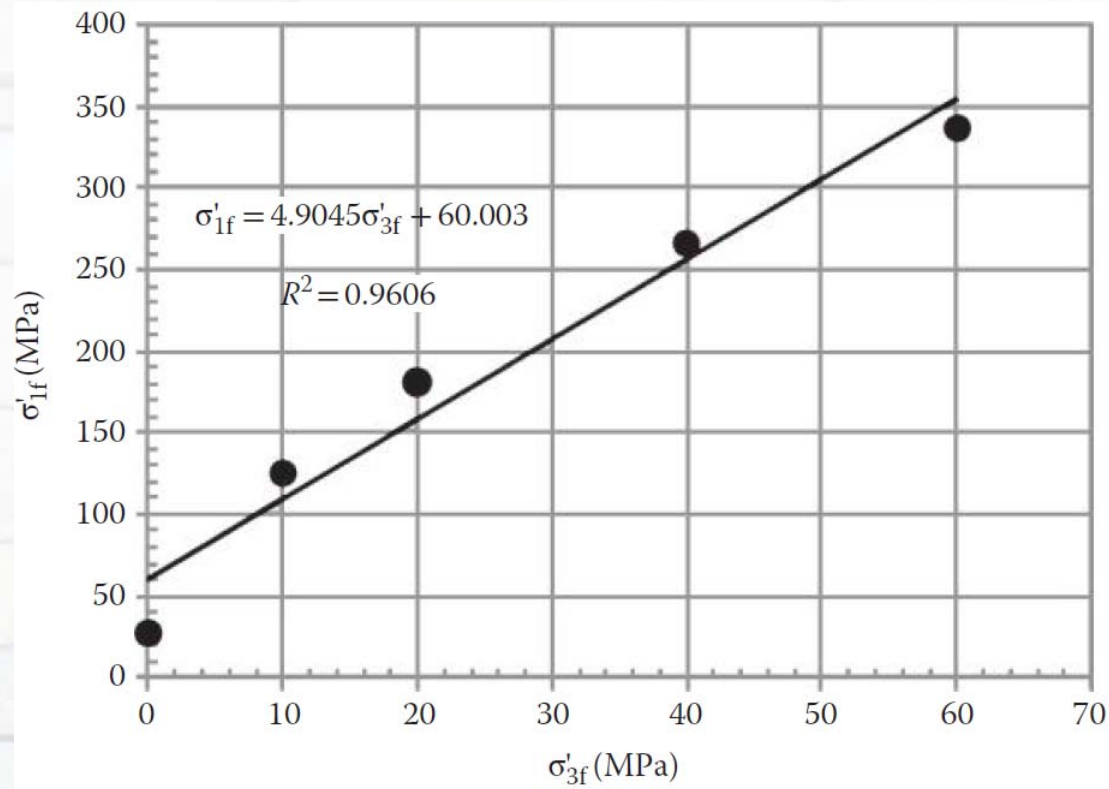
σ'_{3f} (MPa)	0	10	20	40	60
σ'_{1f} (MPa)	27.3	125.6	181.3	266.5	336.9

- Draw the Mohr-Coulomb failure envelope in σ'_1 - σ'_3 space;
- Estimate the effective strength parameters c' and ϕ' and compare them with the experimental data.
- Calculate the global rock mass strength, σ'_{cm} and compare it with the uniaxial compressive strength of the rock mass σ_{cm} determined in Example 5.

Mohr-Coulomb parameters for rock mass

Example 6 - Solution

$$\sigma'_1 = \tan^2\left(45 + \frac{\phi'}{2}\right) \sigma'_3 + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$



Deformation modulus of rock mass

- The **Young's modulus of the intact rock (E_i)** is derived from uniaxial compression tests on the intact cores.
- The **rock mass Young's modulus (E_m)** is determined from empirical correlations.

- Determining E_m from the Q -value:

$$E_m = 25 \log Q \quad (\text{for } Q > 1) \quad \text{Grimstad and Barton (1993)}$$

- Determining E_m from the RMR :

$$E_m (\text{GPa}) = 2 RMR - 100 \quad (\text{for } RMR > 55) \quad \text{Bieniawski (1978)}$$

$$E_m (\text{GPa}) = 10^{\frac{RMR-10}{40}} \quad \text{Serafim and Pereira (1983)}$$

Deformation modulus of rock mass

➤ Determining E_m from the GSI :

$$E_m (\text{GPa}) = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_{ci}}{100}} \times 10^{\frac{GSI-10}{40}} \quad (\text{for } \sigma_{ci} < 100 \text{ MPa}) \quad \text{Hoek et al. (2002)}$$

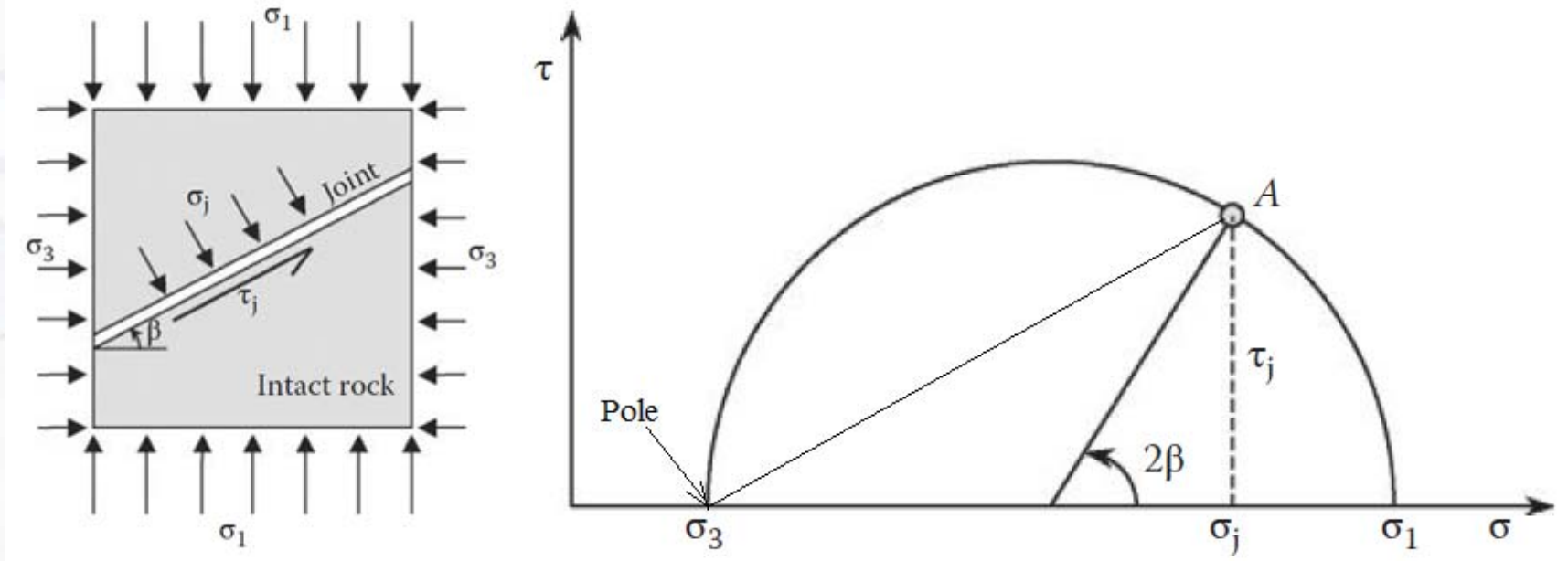
$$E_m (\text{GPa}) = \left(1 - \frac{D}{2}\right) \times 10^{\frac{GSI-10}{40}} \quad (\text{for } \sigma_{ci} > 100 \text{ MPa}) \quad \text{Hoek et al. (2002)}$$

$$E_m (\text{GPa}) = 100 \left(\frac{1 - D/2}{1 + e^{(75+25D-GSI)/11}} \right) \quad \text{Hoek and Diederichs (2006)}$$

$$E_m = E_i \left(0.02 + \frac{1 - D/2}{1 + e^{(60+15D-GSI)/11}} \right) \quad \text{Hoek and Diederichs (2006)}$$

Rock mass with a single plane of weakness

- The **discontinuity planes are often weaker** than the intact rock with a lower cohesion and friction angle.

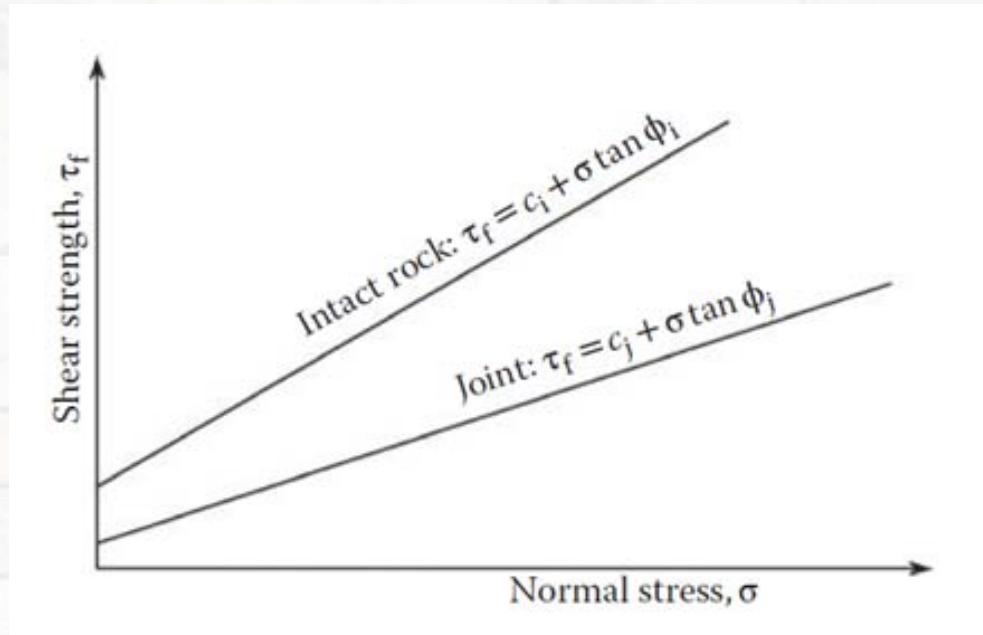
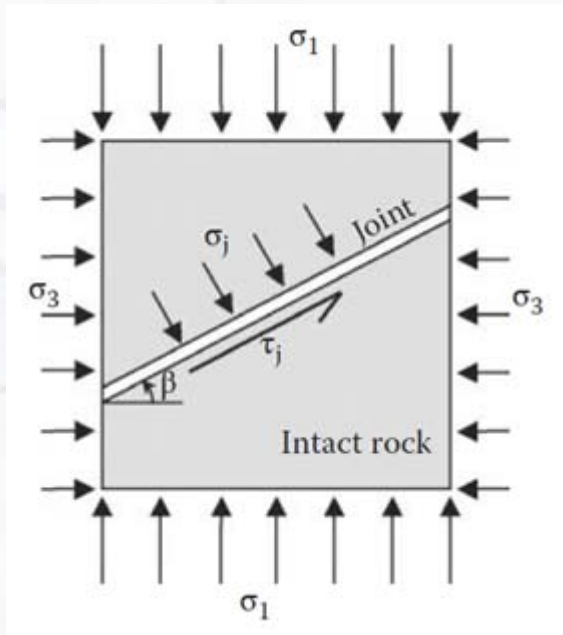


$$\sigma_j = \sigma_3 + \left(\frac{\sigma_1 - \sigma_3}{2} \right) (1 + \cos 2\beta)$$

$$\tau_j = \left(\frac{\sigma_1 - \sigma_3}{2} \right) \sin 2\beta$$

Rock mass with a single plane of weakness

- For **failure to take place along a joint**, the shear stress should reach the **shear strength** given by:



$$\tau_j = c_j + \sigma_j \tan \phi_j$$

$$\sigma_1 - \sigma_3 = \frac{2(c_j + \sigma_3 \tan \phi_j)}{\sin 2\beta \left(1 - \frac{\tan \phi_j}{\tan \beta} \right)}$$

Rock mass with a single plane of weakness

Example 7

A large extent of rock mass has a single plane of discontinuity, where the aperture is filled. The shear strength parameters for the fill material are $c = 4.0$ MPa and $\phi = 34^\circ$. For $\sigma_3 = 3$ MPa, calculate σ_1 for different values of β and plot the variation of σ_1 versus β .

Rock mass with a single plane of weakness

Example 7 - Solution

Minor principal stress, σ_3	β	β	Major principal stress, σ_1	$\beta = \phi$
(MPa)	(degrees)	(rad)	(MPa)	(degrees)
3.00			0.000	34.0
3.00	36	0.628	179.868	34.0
3.00	37	0.646	122.475	34.0
3.00	38	0.663	93.847	34.0
3.00	39	0.681	76.727	34.0
3.00	40	0.698	65.364	34.0
3.00	42	0.733	51.283	34.0
3.00	47	0.820	35.550	34.0
3.00	52	0.908	29.248	34.0
3.00	57	0.995	26.466	34.0
3.00	62	1.082	25.657	34.0
3.00	67	1.169	26.466	34.0
3.00	70	1.222	27.840	34.0
3.00	75	1.309	32.409	34.0
3.00	80	1.396	42.978	34.0
3.00	83	1.449	57.294	34.0
3.00	85	1.484	76.727	34.0
3.00	86	1.501	93.847	34.0
3.00	87	1.518	122.475	34.0
3.00	88	1.536	179.868	34.0

$$\sigma_1 - \sigma_3 = \frac{2(c_j + \sigma_3 \tan \phi_j)}{\sin 2\beta \left(1 - \frac{\tan \phi_j}{\tan \beta}\right)}$$

$c = 4.00$ MPa
 $\phi = 34.00$ degrees
 $\phi = 0.5934$ rad

